

Quantising the electromagnetic field near a semi-transparent mirror

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This paper uses a quantum image detector method to model light scattering on flat surfaces which range from perfectly-reflecting to highly-absorbing mirrors. Instead of restricting the Hilbert space of the electromagnetic field to a subset of modes, we double its size. In this way, the possible exchange of energy between the electromagnetic field and the mirror surface can be taken into account. Finally, we derive the spontaneous decay rate of an atom in front of a semi-transparent mirror as a function of its transmission and reflection rates. Our approach reproduces well-known results, like free-space decay and the sub and super-radiance of an atom in front of a perfectly-reflecting mirror, and paves the way for the modelling of more complex systems with a wide range of applications in quantum technology.

I. INTRODUCTION

The question of how to model the emission of light from atomic systems is older than quantum physics itself. For example, Planck's seminal paper on the spectrum of black body radiation [1] is what eventually led to the discovery of quantum physics. Nowadays, we routinely use quantum optical master equations [2, 3] or a quantum jump approach [4–6] to analyse the dynamics of atomic systems with spontaneous photon emission. For example, the spontaneous decay rate of a two-level atom with ground state $|1\rangle$ and excited state $|2\rangle$ equals

$$\Gamma_{\text{free}} = \frac{e^2 \omega_0^3 \|\mathbf{D}_{12}\|^2}{3\pi \hbar \epsilon c^3} \quad (1)$$

in a medium with permittivity ϵ . Here e is the charge of a single electron and c denotes the speed of light in the medium. Moreover, ω_0 denotes the frequency and \mathbf{D}_{12} is the dipole moment of the 1-2 transition.

Other authors studied the spontaneous photon emission of atomic systems in front of a perfectly-reflecting mirror [7–20]. This is usually done by imposing the boundary condition of a vanishing electric field amplitude along the mirror surface, thereby reducing the available state space of the electromagnetic field in front of the mirror to a subset of photon modes. Compared to modelling an electromagnetic field without boundary conditions, only half of the Hilbert space is taken into account. As a result, the spontaneous decay rate Γ_{mirr} of an atom in front of a perfect mirror differs strongly from the free-space decay rate Γ_{free} in Eq. (1), when the atom-mirror distance x is of the same order of magnitude as the wavelength λ_0 of the emitted light. Although the effect of the mirror is very short range, the sub and super-radiance of atomic systems near perfect mirrors has already been verified experimentally [21].

The canonical quantisation of the electromagnetic field in the presence of a semi-transparent mirror or dielectric medium is less straightforward [22–30]. One way to model light scattering in this case is to proceed as in the case of a perfect mirror and to consider a subset of incoming and outgoing photon modes which are the

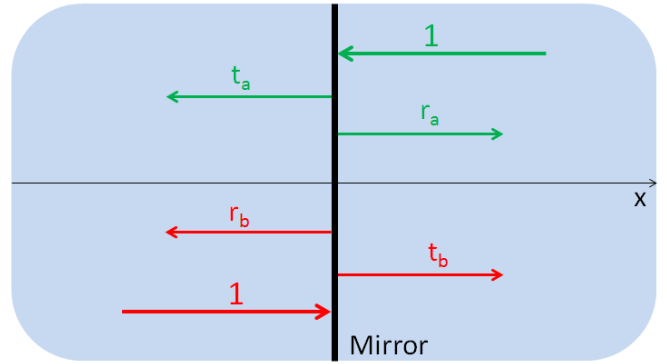


FIG. 1. [Colour online] Schematic view of a semi-transparent mirror with light incident from both sides with finite transmission and reflection rates. Depending on the direction of the incoming light, we denote these rates t_a , r_a and t_b , r_b . For simplicity we assume that the medium on both sides of the mirror is the same. The possible absorption of light in the mirror surface is explicitly taken into account.

stationary solutions of Maxwell's equations [31]. However, the stationary eigenmodes of semi-transparent mirrors are in general no longer pairwise orthogonal [32]. An alternative and purely phenomenological approach to the modelling of the electromagnetic field near a semi-transparent mirror is the so-called input-output formalism [33–35]. Moreover, when modelling the transmission of single photons through optical elements like beamsplitters, we usually employ transition matrices [36, 37]. The consistency and relationship between these different approaches remains an open question [38].

In this paper, we use the same notion of photons as in free space [39] and quantise the electromagnetic field near a semi-transparent mirror with finite transmission and reflection rates in an alternative way. Depending on the origin of the incoming wave packet, we denote these rates t_a , r_a and t_b , r_b , as illustrated in Fig. 1. The main difference between our approach and previous field quantisation schemes is that the energy of the mirror surface, i.e. the energy of the mirror images, is explicitly taken into account. For example, as we shall see below, when

placing a wave packet on one side of a perfect mirror, half of the energy of the system belongs to the original wave packet and the other half belongs to its mirror image. In general, there is a difference between the system Hamiltonian H_{sys} and the Hamiltonian H_{field} , which describes the energy of the electromagnetic field surrounding a semi-transparent mirror. When an incoming wave packet comes in contact with the mirror, energy can flow from the field onto the mirror. Moreover, the squares of the electric field amplitudes of reflected and transmitted wave do not have to add up to one, meaning

$$t_a^2 + r_a^2 \leq 1 \quad \text{and} \quad t_b^2 + r_b^2 \leq 1. \quad (2)$$

The possible absorption of light in the mirror surface is in the following taken into account. Our only assumption is that the mirror surface does not alter the coherent properties of the incoming light. It only reduces the amplitude of incoming wave packets.

Before quantising the electromagnetic field, we use classical electrodynamics to analyse light scattering on flat surfaces, which range from perfectly-reflecting to highly-absorbing mirrors [40]. Doing so, we see that one way of predicting the dynamics of wave packets is to evolve them exactly as in free space. However, the presence of the mirror changes, how and where electric field amplitudes are measured. Detectors now observe superpositions of electric field amplitudes that are associated with incoming, reflected and transmitted waves. Subsequently taking this into account when quantising the electromagnetic field of a two-sided semi-transparent mirror, we find that its system Hamiltonian H_{sys} is the sum of two free space Hamiltonians. Moreover, the electric field observable is a superposition of free space observables with normalisation factors that depend only on the transmission and reflection rates of the mirror. One can easily check that our approach is consistent with Maxwell's equations and that it reproduces the correct long-term dynamics.

When using our field quantisation scheme to derive the master equations of a two-level atom in front of a semi-transparent mirror, we find that the mirror strongly alters the atomic decay rate Γ_{mirr} for small atom-mirror distances. As we shall see below, the mirror has exactly the same effect as a dipole-dipole interaction between the original atom and its mirror image in free space. When $r_a, r_b = 1$ and $t_a, t_b = 0$, our approach reproduces the expected sub and super-radiance of an atom in front of a perfectly-reflecting mirror. Depending on the atom-mirror distance x and the orientation of the atomic dipole \mathbf{D}_{12} , the light radiating from the atom interferes constructively or destructively [7, 14, 17, 19]. Moreover, for $r_a, r_b = 0$ and $t_a, t_b = 1$, our predicted spontaneous decay rate Γ_{mirr} simplifies to its free space value Γ_{free} in Eq. (1).

We expect that the results derived in this paper find a wide range of applications in quantum technology. These range from quantum metrology [41] to the processing of information in coherent cavity-fibre networks [42]. While

it is well known how to quantise the electromagnetic field inside an almost perfect optical cavity [43], modelling more realistic configurations, like two-sided optical resonators with off-resonant laser driving, remains challenging [30].

There are five sections in this paper. In Sec. II, we use classical electrodynamics to map the scattering of light on flat surfaces onto analogous free space scenarios. In Sec. III, we follow the ideas of Ref. [39] to quantise the electromagnetic field in free space. Afterwards, we obtain expressions for the quantum observables of the electromagnetic field which are consistent with classical electrodynamics. In Sec. IV, we test our field quantisation scheme by deriving the master equation of an atom in front of a semi-transparent model. Demanding that an atom at a relatively large distance from the mirror surface decays with the same spontaneous decay rate as an atom in free space, allows us to determine two previously unknown normalisation factors, η_a and η_b , of the electric field observable $\mathbf{E}_{\text{mirr}}(\mathbf{r})$. Finally, we review our findings in Sec. V. Some more mathematical details can be found in Apps. A-D.

II. CLASSICAL LIGHT SCATTERING

In this section, we review light scattering in classical electrodynamics [40]. First we have a closer look at light propagation in free space. Subsequently, we describe the reflection of light by a perfect and by a semi-transparent mirror. For simplicity, we restrict ourselves to wave packets that approach the mirror surface from an orthogonal direction and consider only light propagation in one dimension. Moreover, in the following only a single polarisation is taken into account.

A. Free space

In free space, where there is no restriction to the propagation of light, we simply describe the dynamics of the electromagnetic field by Maxwell's equations. In a medium with permittivity ε , permeability μ and in the absence of any charges or currents, these are given by

$$\begin{aligned} \nabla \cdot \mathbf{E}(\mathbf{r}, t) &= 0, \quad \nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}, \\ \nabla \cdot \mathbf{B}(\mathbf{r}, t) &= 0, \quad \nabla \times \mathbf{B}(\mathbf{r}, t) = \varepsilon \mu \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t}. \end{aligned} \quad (3)$$

Here, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ denote the electric and the magnetic field vectors at position \mathbf{r} and at a time t , respectively. The general solutions to Eq. (3) are superpositions of travelling waves with wave vectors \mathbf{k} , positive frequencies ω and polarisations λ [40].

For simplicity, we consider in the following only travelling waves, which propagate along the x axis such that $\mathbf{k} = (k, 0, 0)$, and allow only for a single polarisation λ . As illustrated in Fig. 2, we fix the direc-

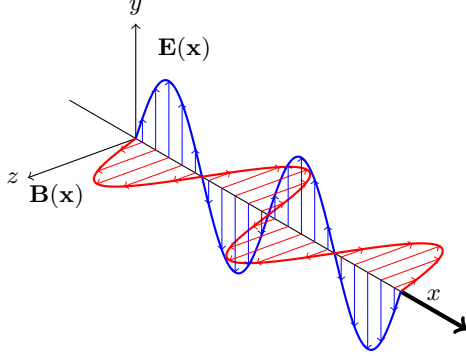


FIG. 2. [Colour online] Schematic view of a right-travelling wave. In this section, we consider only a single polarisation. Without restrictions, we assume that the wave vector points in this case in the positive x direction, while the electric field points in the direction of the y axis and the magnetic field points in the direction of the z axis, which is consistent with the so-called right-hand rule. In contrast to this, the wave vector of a left-travelling wave points in the negative x direction and its magnetic field points in the negative z direction.

tion of the electric and magnetic field vectors such that $\mathbf{E}(\mathbf{r}, t) = (0, E(x, t), 0)$ and $\mathbf{B}(\mathbf{r}, t) = (0, 0, B(x, t))$. For these fields, Maxwell's equations simplify to

$$\begin{aligned}\partial_x E(x, t) &= \pm \partial_t B(x, t), \\ \partial_x B(x, t) &= \pm \varepsilon \mu \partial_t E(x, t).\end{aligned}\quad (4)$$

If we choose the positive directions of the field such that they are consistent with the so-called right-hand rule of electrodynamics, then the plus signs apply to waves travelling in the positive x direction and the minus signs apply to waves travelling in the negative x direction. In the following, we refer to those waves as right- and left-travelling waves. Eliminating the magnetic field from Eq. (4), we find that

$$\partial_x^2 E(x, t) = \varepsilon \mu \partial_t^2 E(x, t), \quad (5)$$

which is the well-known wave equation for the propagation of the electric field in free space.

The general solution $E_{\text{free}}(x, t)$ of this one-dimensional wave equation can always be written as [40]

$$E_{\text{free}}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \tilde{E}_{\text{free}}(k) e^{i(kx - \omega t)} + \text{c.c.}, \quad (6)$$

where the $\tilde{E}_{\text{free}}(k)$ are complex amplitudes and where ω obeys the Kramers-Kronig relation,

$$\omega = |k|c \quad (7)$$

with $c = 1/\sqrt{\varepsilon\mu}$ denoting the speed of light. The dependence of Eq. (6) on $|k|x - \omega t$ for positive k and its dependence on $|k|x + \omega t$ for negative k indicates that these wavenumbers belong to right- and to left-travelling waves, respectively.

Suppose $E_{\text{free}}(x, t)$ is known at an initial time $t = 0$. Then we can calculate the field amplitudes $\tilde{E}_{\text{free}}(k)$ via a Fourier transform. Doing so, we find that

$$\tilde{E}_{\text{free}}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx E_{\text{free}}(x, 0) e^{-ikx} + \text{c.c.} \quad (8)$$

Substituting these coefficients into Eq. (6) yields the electric field $E_{\text{free}}(x, t)$ for the given initial state at all times t . For example,

$$E_{\text{free}}(x, 0) = E_0 e^{-(x-x_0)^2/2\sigma^2} e^{ik_0 x} + \text{c.c.} \quad (9)$$

describes a Gaussian wave packet located around x_0 in position and around k_0 in frequency space. In this case, we have

$$\tilde{E}_{\text{free}}(k) = E_0 \sigma e^{-\sigma^2(k-k_0)^2/2} e^{-i(k-k_0)x_0} + \text{c.c.} \quad (10)$$

Fig. 3 shows the corresponding electric field $E_{\text{free}}(x, t)$ for $t = 0$ and for two later times. As one would expect, the Gaussian wave packet moves with constant speed to the left when k_0 is negative.

B. Perfect one-sided mirrors

The surface charges of perfect mirrors move freely and are able to immediately compensate for any non-zero electric field contributions. Hence the electric field $E_{\text{mirr}}(x, t)$ along the mirror surface at $x = 0$ obeys the boundary condition

$$E_{\text{mirr}}(0, t) = 0 \quad (11)$$

at all times t . In addition, the general electric field solution $E_{\text{mirr}}(x, t)$ must obey Maxwell's equations. Suppose a wave packet approaches the mirror from the right. Until reaching the mirror, the wave packet propagates as in free space. However, when reaching the mirror surface, its electric field amplitude becomes negative and the direction of propagation changes. Interference occurs as long as the incoming and the outgoing contributions meet near the mirror surface. Eventually, however, the wave packet regains its initial shape and continues to travel to the right.

The easiest way of taking the boundary condition in Eq. (11) into account when solving Maxwell's equations is to introduce a mirror image [40]. In the following, we review this method. It suggests to write the electric field $E_{\text{mirr}}(x, t)$ on the right side of the mirror as

$$E_{\text{mirr}}(x, t) = [E_{\text{free}}(x, t) - E_{\text{free}}(-x, t)] \Theta(x), \quad (12)$$

where $\Theta(x)$ denotes the Heaviside step function

$$\Theta(x) = \begin{cases} 1 & \text{for } x \geq 0, \\ 0 & \text{for } x < 0. \end{cases} \quad (13)$$

One can easily check that Eq. (12) obeys the boundary condition in Eq. (11) at all times. Moreover, $E_{\text{mirr}}(x, t)$

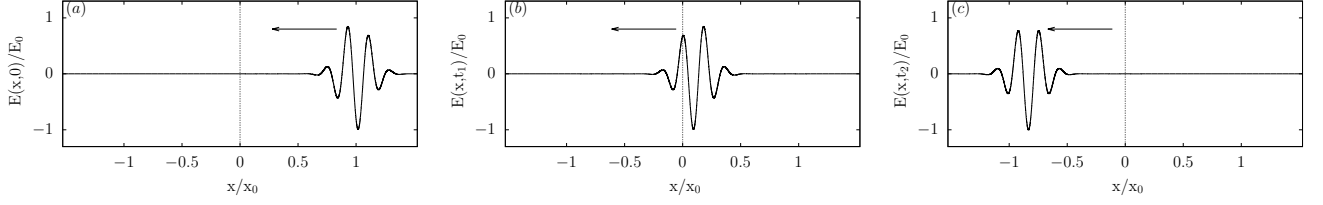


FIG. 3. Electric field amplitude $E_{\text{free}}(x, t)$ of a left-moving Gaussian wave packet in free space at three different times t . The initial state of the wave packet is given in Eq. (9) and its dynamics have been obtained by combining Eqs. (6) and (10) while $k_0 x_0 = -6$, $\sigma = (1/\sqrt{2}) x_0$ with $t_1 = 0.89x_0/c$ and $t_2 = 1.83x_0/c$.

obeys Maxwell's equations, since it is the sum of free-space solutions. Most importantly, we now have a solution for the electric field $E_{\text{mirr}}(x, t)$ on the right-hand side of the mirror, which only requires solving Maxwell's equations in free space.

One way of interpreting the electric field solution in Eq. (12) is to say that the mirror produces a mirror image. The mirror image has the same shape as the original wave packet but its components travel in opposite directions and its amplitude is such that

$$E_{\text{image}}(x, 0) = -E_{\text{free}}(-x, 0). \quad (14)$$

Propagating the initial electric field

$$E_{\text{mirr}}(x, 0) = E_{\text{free}}(x, 0) + E_{\text{image}}(x, 0) \quad (15)$$

freely in time yields exactly the same electric field as Eq. (12) as long as we restrict ourselves to the $x \geq 0$ half space. This is illustrated in Fig. 4. Fig. 4(a)–(c) and Fig. 4(d)–(f) show a left-moving and a right-moving wave packet, respectively, at three different times. The two wave packets cross over $x = 0$ at the same time. Adding the electric field contributions on the right-hand side of the mirror, as done in Fig. 4(g)–(i), reproduces the dynamics of an incoming wave packet that approaches the mirror from the left.

An alternative way of interpreting Eq. (12) is to say that the mirror introduces an *image detector*, while wave packets propagate exactly as in free space. Assuming that wave packets propagate as in the absence of any mirrors ensures that the dynamics of the wave packets satisfies Maxwell's equations. The presence of the image detector takes into account that the mirror changes where and how the electric field is observed. Suppose the image detector measures $-E_{\text{free}}(x, t)$ with $x \leq 0$, while the original detector measures $E_{\text{free}}(x, t)$ with $x \geq 0$. Assuming moreover that the total electric field $E_{\text{mirr}}(x, t)$ seen near the perfect mirror is the sum of the fields seen by both the original detector and the image detector reproduces the general electric field solution in Eq. (12).

C. Perfect two-sided mirrors

Next let us have a closer look at what happens when wave packets approach a two-sided and perfectly reflect-

ing mirror at $x = 0$ from both sides. Doing so is straightforward, since wave packets on the different sides of the mirror never meet and never interfere. Proceeding as above, we find that the electric field amplitude $E_{\text{mirr}}(x, t)$ can now be written as

$$E_{\text{mirr}}(x, t) = \left[E_{\text{free}}^{(a)}(x, t) - E_{\text{free}}^{(a)}(-x, t) \right] \Theta(x) + \left[E_{\text{free}}^{(b)}(x, t) - E_{\text{free}}^{(b)}(-x, t) \right] \Theta(-x) \quad (16)$$

in analogy to Eq. (12). The superscripts (a) and (b) in this equation help to distinguish free space solutions corresponding to opposite sides of the mirror. One can easily check that this general electric field solution satisfies the wave equation in Eq. (5) and the boundary condition in Eq. (11) at all times.

D. Semi-transparent mirrors

Finally, we are ready to solve Maxwell's equations in the presence of a semi-transparent mirror. As illustrated in Fig. 1, we denote the transmission and reflection rates of the mirror t_a , t_b , r_a and r_b . To find the general electric field solution in this case, we proceed as in the previous subsection and write the electric field $E_{\text{mirr}}(x, t)$ as the sum of free-space solutions. The main difference to the case of a two-sided perfect mirror is that the contributions seen by the image detectors now need to be weighted with r_a and t_b , and with t_a and r_b , respectively. Hence we find that

$$E_{\text{mirr}}(x, t) = \left[E_{\text{free}}^{(a)}(x, t) - r_a E_{\text{free}}^{(a)}(-x, t) + t_b E_{\text{free}}^{(b)}(x, t) \right] \Theta(x) + \left[E_{\text{free}}^{(b)}(x, t) - r_b E_{\text{free}}^{(b)}(-x, t) + t_a E_{\text{free}}^{(a)}(x, t) \right] \Theta(-x) \quad (17)$$

in analogy to Eq. (16). Again, one can easily check that this solution is consistent with Maxwell's equations. However, $E_{\text{mirr}}(x, t)$ no longer satisfies the boundary condition in Eq. (11), since semi-transparent mirrors do not have enough surface charges to compensate for all parallel non-zero electric field contributions. As pointed out already in Sec. I, the possible absorption of light in the mirror surface is explicitly taken into account (c.f. Eq. (2)),

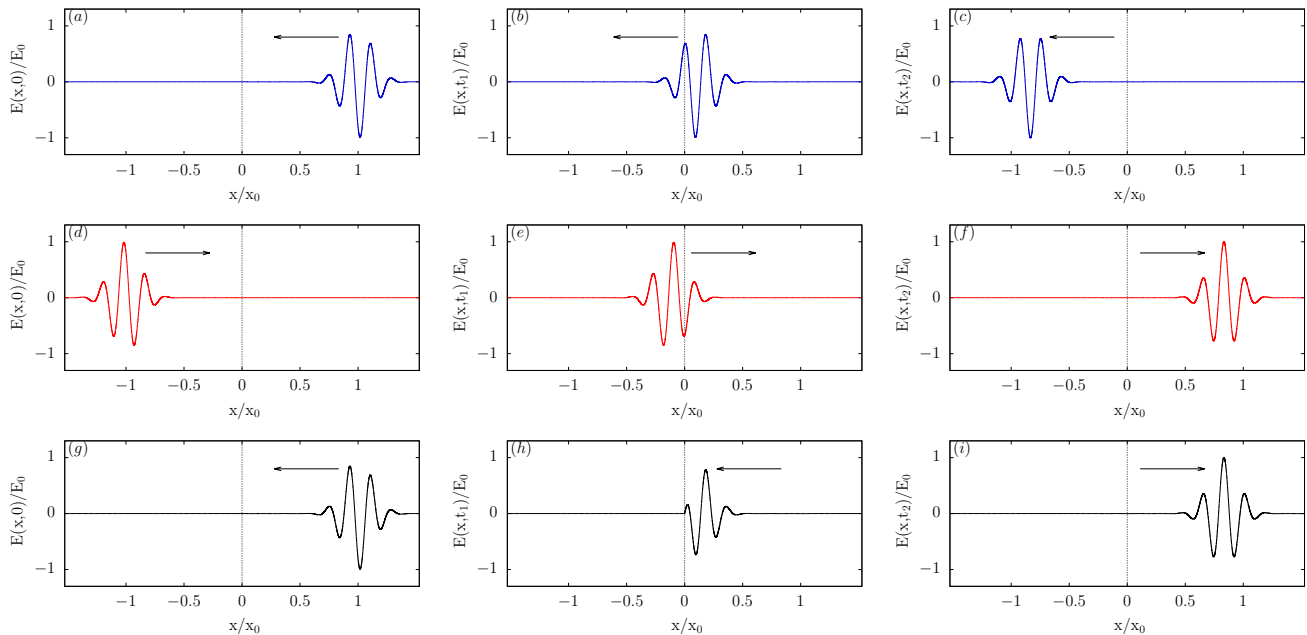


FIG. 4. [Colour online] Plots (a)–(c) and (d)–(f) show a left- and right-travelling wave packet respectively, for the same parameters as in Fig. 3. At $t = 0$, the blue wave packet ((a)–(c)) can be interpreted as a real wave packet, while the red wave packet ((d)–(f)) constitutes its mirror image. When the wave packets cross over at $x = 0$, the red wave packet becomes real, while the blue one becomes the image. Moreover, plots (g)–(i) show the sum of the red and the blue electric field contribution on the right-hand side of the mirror, which evolves like a wave packet approaching a perfectly reflecting mirror.

thereby allowing a portion of the energy of incoming wave packets to be dissipated within the mirror surface. However, we assume here that the absorption does not affect the general shape and coherence properties of the incoming wave packets. It only reduces their amplitude.

III. A QUANTUM IMAGE DETECTOR METHOD TO LIGHT SCATTERING

In Sec. II, we mapped the scattering of light through a semi-transparent mirror onto analogous free-space scenarios. In this section, we use this to derive the electric field observables $\mathbf{E}_{\text{mirr}}(\mathbf{r})$ and the system Hamiltonian H_{sys} for the electromagnetic field near a semi-transparent mirror as a function of its transmission and reflection rates. To do so, we demand that field expectation values evolve as predicted by Maxwell's equations. In addition, certain boundary conditions need to be taken into account. For a perfect mirror, we demand that the electric field amplitude vanishes on the mirror surface. For a semi-transparent mirror, the electric field expectation values of incoming wave packets need to exhibit the same long-term behaviour as in classical physics.

A. Free space

As in Sec. II, we first have a closer look at free space. As in the previous section, we restrict ourselves at first to light propagation in one-dimension and consider only a single polarisation λ . As illustrated in Fig. 2, we assume that the electric and the magnetic field points in the y and the z direction respectively. For simplicity and to avoid having to introduce a certain gauge, we proceed in this subsection as in Ref. [39] to obtain an expression for the free field observables, which we denote $E_{\text{free}}(x)$, $B_{\text{free}}(x)$ and H_{sys} .

The free-space expression for H_{sys} can be deduced easily from experimental observations. Experiments confirm that the electromagnetic field in free space consists of basic energy quanta (photons) with positive and negative wavenumbers k . When measured, photons occur only in integer numbers. This implies that the electromagnetic field is a collection of harmonic oscillator modes with basis states $|n_k\rangle$, where n_k is an integer. Hence, the electromagnetic field in free space can be fully described by tensor product states of the form

$$\bigotimes_{k=-\infty}^{\infty} |n_k\rangle. \quad (18)$$

These energy eigenstates span the Hilbert space \mathcal{H} of the electromagnetic field. As in Sec. II A before, we use positive and negative wavenumbers k for right- and left-moving waves. The frequency ω of a photon in the k -

mode is as usual given by the Kramers-Kronig relation in Eq. (7).

Moreover, experiments show that the energy of a single photon of frequency ω equals $\hbar\omega$. To take this into account, we now introduce the bosonic annihilation and creation operators, a_k and a_k^\dagger , which act on the above introduced Hilbert space \mathcal{H} and obey the bosonic commutator relation

$$[a_k, a_{k'}^\dagger] = \delta(k - k'). \quad (19)$$

Using this notation, the system Hamiltonian H_{sys} can be written as

$$H_{\text{sys}} = \int_{-\infty}^{\infty} dk \hbar\omega a_k^\dagger a_k. \quad (20)$$

Moreover, we know that the observable for the the energy stored inside the electromagnetic field H_{field} equals

$$H_{\text{field}} = \frac{1}{2}A \int_{-\infty}^{\infty} dx \left[\varepsilon E_{\text{free}}(x)^2 + \frac{1}{\mu} B_{\text{free}}(x)^2 \right], \quad (21)$$

where A denotes the area in the y - z plane in which H_{sys} and H_{field} are defined and where $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ denote the free-space observables of the electric and the magnetic field, respectively. In the absence of any mirrors, both Hamiltonians coincide,

$$H_{\text{sys}} = H_{\text{field}}, \quad (22)$$

up to a constant which is known as the zero-point energy of the electromagnetic field. In the following, we neglect this constant, which is of no relevance for the calculations in this paper.

Eqs. (20)–(22) show that $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ must be linear superpositions of a_k and a_k^\dagger . Writing $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ as such superpositions, one can calculate the corresponding coefficients using Ehrenfest's theorem for field observables O ,

$$\frac{d}{dt}\langle O \rangle = -\frac{i}{\hbar}\langle [O, H_{\text{sys}}] \rangle, \quad (23)$$

and demanding that expectation values of $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ evolve according to Maxwell's equations. Doing so and choosing the normalisation of the field operators $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ such that Eq. (22) holds (c.f. App. A for more details), yields

$$\begin{aligned} E_{\text{free}}(x) &= i \int_{-\infty}^{\infty} dk \sqrt{\frac{\hbar\omega}{4\pi\varepsilon A}} e^{ikx} a_k + \text{H.c.}, \\ B_{\text{free}}(x) &= -i\sqrt{\varepsilon\mu} \int_{-\infty}^{\infty} dk \sqrt{\frac{\hbar\omega}{4\pi\varepsilon A}} e^{ikx} a_k \text{sign}(k) \\ &\quad + \text{H.c.} \end{aligned} \quad (24)$$

The factor $\text{sign}(k)$ accounts for the different orientations of the magnetic field amplitudes for left- and right-travelling waves for a given orientation of the electric field.

B. Perfect one-sided mirrors

Next, we consider a perfect one-sided mirror at $x = 0$ with non-zero field components only on its right-hand side. In Sec. II we have seen that, even in this case, wave packets evolve essentially as in free space. However, what changes is how and where electromagnetic field amplitudes are measured. Moreover, we know from experience that a photon of frequency ω has the energy $\hbar\omega$, even in the presence of a mirror. Hence the Hamiltonian H_{sys} of the electromagnetic field in the presence of a perfectly-reflecting mirror equals the free space Hamiltonian H_{sys} in Eq. (20).

Replacing the mirror by image detectors, we find that the electric field seen by a detector on the right-hand side of the mirror is the sum of the field seen by the original detector and the field seen by its respective image detector. Taking this into account, we write the electric field observable $E_{\text{mirr}}(x)$ in front of a perfect mirror surface as a superposition of two free-space solutions. In analogy to Eq. (12), we find that

$$E_{\text{mirr}}(x) = \frac{1}{\eta} [E_{\text{free}}(x) - E_{\text{free}}(-x)] \Theta(x) \quad (25)$$

with $E_{\text{free}}(x)$ as in Eq. (24) and with η being a normalisation factor. To find the corresponding magnetic field amplitude $B_{\text{mirr}}(x)$, we invoke Maxwell's equations. Taking into account that the free-field observables $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ in Eq. (24) are already consistent with Eq. (4), Eq. (25) implies

$$B_{\text{mirr}}(x) = \frac{1}{\eta} [B_{\text{free}}(x) + B_{\text{free}}(-x)] \Theta(x). \quad (26)$$

Magnetic field expectation values do not vanish on the surface of a perfect mirror. One can easily check that the dynamics of the expectation values of $E_{\text{mirr}}(x)$ and $B_{\text{mirr}}(x)$ in Eqs. (25) and (26) are consistent with Maxwell's equations.

The normalisation factor η in Eqs. (25) and (26) takes into account that the creation operators a_k^\dagger now generate plane waves with different electric field amplitudes from those in free space. To determine η , we define the standing-wave photon annihilation operators ξ_k as

$$\xi_k = \frac{1}{\sqrt{2}} (a_k - a_{-k}) \quad \text{with} \quad \xi_{-k} = -\xi_k. \quad (27)$$

Using this notation and combining Eqs. (24)–(26), one can then show that

$$\begin{aligned} E_{\text{mirr}}(x) &= \frac{i}{\eta} \int_{-\infty}^{\infty} dk \sqrt{\frac{\hbar\omega}{2\pi\varepsilon A}} e^{ikx} \xi_k \Theta(x) + \text{H.c.}, \\ B_{\text{mirr}}(x) &= -\frac{i\sqrt{\varepsilon\mu}}{\eta} \int_{-\infty}^{\infty} dk \sqrt{\frac{\hbar\omega}{2\pi\varepsilon A}} e^{ikx} \xi_k \text{sign}(k) \Theta(x) \\ &\quad + \text{H.c.} \end{aligned} \quad (28)$$

Using these field observables to calculate the energy of the electromagnetic field on the right-hand side of the

mirror we find

$$H_{\text{field}} = \frac{1}{2} A \int_0^\infty dx \left[\varepsilon E_{\text{mirr}}(x)^2 + \frac{1}{\mu} B_{\text{mirr}}(x)^2 \right]. \quad (29)$$

Proceeding as described in App. B, we find that H_{sys} and H_{field} are no longer the same. Instead, we now find that

$$H_{\text{field}} = \frac{2}{\eta^2} \int_0^\infty dk \hbar \omega \xi_k^\dagger \xi_k \quad (30)$$

up to a negligible constant. One can easily check that this field Hamiltonian commutes with the system Hamiltonian and that its expectation value is constant in time. The difference between H_{sys} and H_{field} ,

$$H_{\text{mirr}} = H_{\text{sys}} - H_{\text{field}}, \quad (31)$$

is the observable for the energy of the surface charges of the mirror.

Suppose a wave packet with a vanishing electric field amplitude at $x = 0$ approaches the mirror from the right. As illustrated in Fig. 4, this situation is equivalent to having two wave packets travelling in opposite directions in free space. Hence we should find that

$$H_{\text{field}} = H_{\text{mirr}} = \frac{1}{2} H_{\text{sys}} \quad (32)$$

in this case, which implies

$$\eta = \sqrt{2}. \quad (33)$$

Half of the energy of the system is stored in the electromagnetic field. The other half belongs to the image of the incoming wave packet. Hence creating a wave packet with a certain amplitude near a perfectly-conducting metallic surface requires double the energy than creating the same wave packet in free space. Moreover, as we shall see below in Sec. IV, choosing η as suggested in Eq. (33) assures that an atom at a relatively large distance from a perfect mirror has the same spontaneous decay rate as an atom in free space.

Although previous field quantisation schemes for the electromagnetic field in front of a perfect mirror ignore the energy of the mirror surface (see e.g. Refs. [7, 14, 15, 17–19]), their field observables are consistent with our approach. Without loss of generality, one can write any wave packet on the right-hand side of a perfect one-sided mirror such that only the ξ_k photon modes of the electromagnetic field become populated. When this applies, H_{field} in Eq. (30) and the system Hamiltonian H_{sys} in Eq. (20) become the same. Moreover, the field observables $E_{\text{mirr}}(x)$ and $B_{\text{mirr}}(x)$ in Eq. (28) depend only on ξ_k -mode photon operators. Because of Eq. (33), there is therefore no difference between the field operators that we derive here and the field observables obtained when quantising the ξ_k modes of the electromagnetic field as in free space. However, in this paper, we do not restrict ourselves to the subset of ξ_k photon modes and allow for the full range of possible initial states.

C. Perfect two-sided mirrors

As pointed out in Sec. II C, wave packets on opposite sides of a perfectly-reflecting two-sided mirror never meet. Therefore, all we need to do model this case is to quantise the electromagnetic field on each side of the mirror separately. To do so, we replace the Hilbert space \mathcal{H} that we introduced in Sec. III A by the tensor product of two free-space Hilbert spaces $\mathcal{H}^{(a)}$ and $\mathcal{H}^{(b)}$,

$$\mathcal{H} \rightarrow \mathcal{H}^{(a)} \otimes \mathcal{H}^{(b)}. \quad (34)$$

Denoting the photon annihilation operators of the different sides of the mirror by a_k and b_k , respectively, the system Hamiltonian H_{sys} can be written as

$$H_{\text{sys}} = \int_{-\infty}^{\infty} dk \hbar \omega \left[a_k^\dagger a_k + b_k^\dagger b_k \right]. \quad (35)$$

Moreover, in analogy to Eqs. (25) and (26), the electric field observable $E_{\text{mirr}}(x)$ in front of a two-sided perfect mirror equals

$$E_{\text{mirr}}(x) = \frac{1}{\sqrt{2}} \left[E_{\text{free}}^{(a)}(x) - E_{\text{free}}^{(a)}(-x) \right] \Theta(x) + \frac{1}{\sqrt{2}} \left[E_{\text{free}}^{(b)}(x) - E_{\text{free}}^{(b)}(-x) \right] \Theta(-x). \quad (36)$$

The superscripts indicate the Hilbert space on which the respective free-space electric field observable is defined. As in Sec. II C, $^{(a)}$ and $^{(b)}$ refer to the right and the left half space of the mirror, respectively.

D. Semi-transparent mirrors

Generalising the above field quantisation scheme to semi-transparent mirrors is now straightforward. As in the previous subsection, we consider a tensor product Hilbert space $\mathcal{H}^{(a)} \otimes \mathcal{H}^{(b)}$ with photon annihilation operators a_k and b_k , and assume that the system Hamiltonian H_{sys} is as defined in Eq. (35). Moreover, in close analogy to Eq. (17), we write the electric field observable $E_{\text{mirr}}(x)$ in front of the semi-transparent mirror as a superposition of free-space contributions. Doing so, we find that

$$E_{\text{mirr}}(x) = \left[\frac{1}{\eta_a} E_{\text{free}}^{(a)}(x) - \frac{r_a}{\eta_a} E_{\text{free}}^{(a)}(-x) + \frac{t_b}{\eta_b} E_{\text{free}}^{(b)}(x) \right] \Theta(x) + \left[\frac{1}{\eta_b} E_{\text{free}}^{(b)}(x) - \frac{r_b}{\eta_b} E_{\text{free}}^{(b)}(-x) + \frac{t_a}{\eta_a} E_{\text{free}}^{(a)}(x) \right] \Theta(-x) \quad (37)$$

with η_a and η_b being (real) normalisation factors. One can easily check that this observable is consistent with Maxwell's equations. As in classical physics, $E_{\text{mirr}}(x)$ no longer vanishes at $x = 0$. Instead, the individual terms of the above operator have been weighted such that an incoming wave packet turns eventually into two

outgoing wave packets with their respective electric field amplitudes accordingly renormalised.

In the case of a perfect two-sided mirror we have maximum reflection ($r_a, r_b = 1$) and zero transmission ($t_a, t_b = 0$). Indeed, $E_{\text{mirr}}(x)$ in Eq. (37) simplifies to $E_{\text{mirr}}(x)$ in Eq. (36) in this case, if we choose $\eta_a = \eta_b = \sqrt{2}$. Moreover, in free space, we have zero reflection ($r_a, r_b = 0$) and maximum transmission ($t_a, t_b = 1$). In this case, $E_{\text{mirr}}(x)$ in Eq. (37) should simplify to $E_{\text{free}}(x)$ in Eq. (24). This applies when we replace the photon annihilation operators a_k in Eq. (24) by annihilation operators c_k with

$$c_k = \frac{a_k}{\eta_a} + \frac{b_k}{\eta_b} \quad \text{with} \quad \frac{1}{\eta_a^2} + \frac{1}{\eta_b^2} = 1. \quad (38)$$

To find η_a and η_b , we need to specify not only the transmission and reflection rates of the mirror surface. We also need to take the properties of the medium which contains the electromagnetic field on either side of the mirror into account. To do so, Sec. IV analyses the spontaneous emission of an atom in front of a semi-transparent mirror. Demanding that its spontaneous decay rate Γ_{mirr} simplifies to the free-space decay rate Γ_{free} in Eq. (1) for relatively large atom-mirror distances ultimately yields expressions for η_a and η_b (c.f. Eq. (68)).

The transmission of light through a mirror surface results in general in the loss of energy from the electromagnetic field. As pointed out in Sec. II, absorption of light in the mirror surface is already built into our model. In the above quantum model, the energy of the electromagnetic field and the mirror surface, i.e. the expectation value of the system Hamiltonian H_{sys} in Eq. (35), is conserved. However, the expectation value of the electromagnetic field Hamiltonian H_{field} ,

$$H_{\text{field}} = \frac{1}{2} A \int_{-\infty}^{\infty} dx \left[\varepsilon E_{\text{mirr}}(x)^2 + \frac{1}{\mu} B_{\text{mirr}}(x)^2 \right], \quad (39)$$

can change in time. In general, one can show that

$$[H_{\text{field}}, H_{\text{sys}}] \neq 0, \quad (40)$$

which implies a continuous exchange of energy between the electromagnetic field and the surface charges of the mirror. For example, suppose a wave packet approaches the mirror from the right. After a sufficiently long time, this wave packet turns into two wave packets: one on the left-hand side of the mirror with its electric and magnetic field amplitudes $E_{\text{mirr}}(x)$ and $B_{\text{mirr}}(x)$ reduced by a factor t_a , and one on the right with $E_{\text{mirr}}(x)$ and $B_{\text{mirr}}(x)$ reduced by a factor r_a . As one can see from Eq. (39), this implies a reduction of the energy stored inside the electromagnetic field by a factor $r_a^2 + t_a^2$ which can be smaller than one (c.f. Eq. (2)).

E. Generalisation to three dimensions and two polarisations

Finally, we generalise the above field quantisation scheme to three dimensions. Instead of wavenumbers k ,

we now have to consider three-dimensional wave vectors \mathbf{k} . For each \mathbf{k} , there are two possible independent directions of the electric field, i.e. two different polarisations $\lambda = 1$ and $\lambda = 2$. In analogy to Eq. (7), the frequency ω associated with each wave vector \mathbf{k} equals

$$\omega = \|\mathbf{k}\| c. \quad (41)$$

However, apart from having to consider a much larger number of degrees of freedom, there are hardly any differences between quantising the electromagnetic field in one and in three dimensions.

1. Free space

In the absence of any mirrors, the observable of the electric field $\mathbf{E}_{\text{free}}(\mathbf{r})$ at position \mathbf{r} equals [39]

$$\begin{aligned} \mathbf{E}_{\text{free}}(\mathbf{r}) = & \frac{i}{(2\pi)^{3/2}} \sum_{\lambda=1,2} \int_{\mathbb{R}^3} d^3\mathbf{k} \sqrt{\frac{\hbar\omega}{2\varepsilon}} e^{i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda} \\ & + \text{H.c.} \end{aligned} \quad (42)$$

Here $a_{\mathbf{k}\lambda}$ is the photon annihilation operator of the (\mathbf{k}, λ) photon mode and obeys the commutator relation

$$[a_{\mathbf{k}\lambda}, a_{\mathbf{k}'\lambda'}^\dagger] = \delta_{\lambda\lambda'} \delta^3(\mathbf{k} - \mathbf{k}'). \quad (43)$$

The normalised polarisation vectors $\hat{\mathbf{e}}_{\mathbf{k}\lambda}$ in Eq. (42) are pairwise orthogonal and $\hat{\mathbf{e}}_{\mathbf{k}\lambda} \cdot \mathbf{k} = 0$ for all \mathbf{k} . Moreover, the Hamiltonian H_{sys} of the electromagnetic field in three dimensions equals [39]

$$H_{\text{sys}} = \sum_{\lambda=1,2} \int_{\mathbb{R}^3} d^3\mathbf{k} \hbar\omega_k a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} \quad (44)$$

in analogy to Eq. (20). As in Sec. III A, we assume in the following that left-travelling waves have wave vectors \mathbf{k} with negative x components, while right-moving waves have wave vectors \mathbf{k} with positive x components.

2. Semi-transparent mirrors

When introducing a semi-transparent mirror in the $x = 0$ plane, we again double the Hilbert space \mathcal{H} compared to free-space. Denoting the photon annihilation operators in each Hilbert space by $a_{\mathbf{k}\lambda}$ and $b_{\mathbf{k}\lambda}$, the system Hamiltonian H_{sys} of the electromagnetic field and the mirror surface is now given by

$$H_{\text{sys}} = \sum_{\lambda=1,2} \int_{\mathbb{R}^3} d^3\mathbf{k} \hbar\omega \left[a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} + b_{\mathbf{k}\lambda}^\dagger b_{\mathbf{k}\lambda} \right] \quad (45)$$

in analogy to Eq. (35). To obtain the observable $\mathbf{E}_{\text{mirr}}(\mathbf{r})$ of the electric field at position \mathbf{r} , we use again the above introduced quantum image detector method. More concretely, we assume in the following that a detector at a position $\mathbf{r} = (x, y, z)$ observes light arriving directly

at the detector which originates either from the same or from the other side of the mirror. In addition, the detector measures the electric field amplitude at a position $\tilde{\mathbf{r}}$ with

$$\tilde{\mathbf{r}} = (-x, y, z). \quad (46)$$

This final field contribution needs to be multiplied by r_a or r_b as appropriate. Moreover, its y and z components need to be multiplied with -1 , while the x component of the electric field remains the same, before being added to the above mentioned field contributions. Taking this into account, we find that

$$\begin{aligned} \mathbf{E}_{\text{mirr}}(\mathbf{r}) &= \left[\frac{1}{\eta_a} \mathbf{E}_{\text{free}}^{(a)}(\mathbf{r}) - \frac{r_a}{\eta_a} \tilde{\mathbf{E}}_{\text{free}}^{(a)}(\tilde{\mathbf{r}}) + \frac{t_b}{\eta_b} \mathbf{E}_{\text{free}}^{(b)}(\mathbf{r}) \right] \Theta(x) \\ &+ \left[\frac{1}{\eta_b} \mathbf{E}_{\text{free}}^{(b)}(\mathbf{r}) - \frac{r_b}{\eta_b} \tilde{\mathbf{E}}_{\text{free}}^{(b)}(\tilde{\mathbf{r}}) + \frac{t_a}{\eta_a} \mathbf{E}_{\text{free}}^{(a)}(\mathbf{r}) \right] \Theta(-x) \end{aligned} \quad (47)$$

which generalises Eq. (37) to field propagation in three dimensions. Here $\tilde{\mathbf{E}}_{\text{free}}^{(a)}(\tilde{\mathbf{r}})$ and $\tilde{\mathbf{E}}_{\text{free}}^{(b)}(\tilde{\mathbf{r}})$ are defined such that they differ from $\mathbf{E}_{\text{free}}^{(a)}(\tilde{\mathbf{r}})$ and $\mathbf{E}_{\text{free}}^{(b)}(\tilde{\mathbf{r}})$, respectively, only by the sign of their x -component.

IV. THE MASTER EQUATION OF AN ATOM NEAR A SEMI-TRANSPARENT MIRROR

In the limit of relatively large atom-mirror distances x , we expect that the spontaneous decay rate Γ_{mirr} of the atom coincides with its spontaneous free-space decay rate Γ_{free} in Eq. (1). In the following, we impose this condition to calculate the normalisation factors η_a and η_b in Eq. (47) as a function of the reflection and transmission rates r_a , r_b , t_a and t_b of the semi-transparent mirror. The only assumption made in this section in addition to standard quantum optical approximations is that the atom-mirror distance does not become so large that delay terms have to be taken into account [18]. The dynamics of the atom should hence remain Markovian.

A. General derivation

To derive the quantum optical master equations of an atom in front of a semi-transparent mirror, we now follow the approach of Ref. [3] and first identify the operators that need calculating. Our starting point is the Hamiltonian H , which describes the energy of the atom, the free radiation field, the mirror surface and their respective interactions,

$$H = H_{\text{atom}} + H_{\text{sys}} + H_{\text{int}}. \quad (48)$$

When going into the interaction picture with respect to the free Hamiltonian

$$H_0 = H_{\text{atom}} + H_{\text{sys}}, \quad (49)$$

the above Hamiltonian transforms into the interaction Hamiltonian

$$H_I(t) = U_0^\dagger(t, 0) H_{\text{int}} U_0(t, 0). \quad (50)$$

The concrete form of this Hamiltonian can be found in the next subsection.

In addition to this interaction, we need to take the effect of the environment, like the walls of the laboratory, into account [44]. To do so, we assume in the following that the environment constantly resets the free radiation field constantly into its environmentally preferred state [3, 45]. At room temperature and in the optical regime, this state coincides essentially with the vacuum state $|0\rangle$ of the electromagnetic field. Hence the density matrix $\rho_I(t)$ of atom and field is in general of the form

$$\rho_I(t) = \rho_{\text{AI}}(t) \otimes |0\rangle\langle 0|. \quad (51)$$

Here $\rho_{\text{AI}}(t)$ denotes the density matrix of the atom at time t in the interaction picture. Evolving $\rho_I(t)$ with the above interaction Hamiltonian for a short time interval Δt , we find that

$$\rho_I(t + \Delta t) = U_I(t + \Delta t, t) \rho_I(t) U_I^\dagger(t + \Delta t, t). \quad (52)$$

Usually, this time evolution is followed by a rapid resetting of the field back into its vacuum state. Taking this into account and resetting the field on a time scale that is much shorter than Δt without changing the statistical properties of the atom, we find that the density matrix of the atom equals

$$\begin{aligned} \rho_{\text{AI}}(t + \Delta t) &= \text{Tr}_{\text{field}} \left(U_I(t + \Delta t, t) |0\rangle \rho_{\text{AI}}(t) \langle 0| U_I^\dagger(t + \Delta t, t) \right) \end{aligned} \quad (53)$$

at $t + \Delta t$. In the following, we summarise the coarse-grained dynamics implied by this equation by an analogous differential equation without coarse graining, i.e. the atomic master equation

$$\dot{\rho}_{\text{AI}}(t) = \frac{1}{\Delta t} (\rho_{\text{AI}}(t + \Delta t) - \rho_{\text{AI}}(t)). \quad (54)$$

When calculating the terms on the right hand side of this equation, we need to consider a relatively short time interval Δt . However, Δt should not be too short either, to allow for an efficient transfer of atomic excitation into the electromagnetic field [3, 4].

In the following, we use second-order perturbation theory to calculate $U_I(t + \Delta t, t)$, which implies

$$\begin{aligned} U_I(t + \Delta t, t) &= 1 - \frac{i}{\hbar} \int_t^{t+\Delta t} dt' H_I(t') \\ &- \frac{1}{\hbar^2} \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' H_I(t') H_I(t''). \end{aligned} \quad (55)$$

As we shall see below, keeping only terms that are of first order in Δt , one can show that $\dot{\rho}_{\text{AI}}(t)$ evolves in general

according to an equation of the form

$$\dot{\rho}_{\text{AI}}(t) = -\frac{i}{\hbar} \left[H_{\text{cond}} \rho_{\text{AI}}(t) - \rho_{\text{AI}}(t) H_{\text{cond}}^\dagger \right] + \mathcal{L}(\rho_{\text{AI}}(t)) \quad (56)$$

with the so-called conditional Hamiltonian H_{cond} and the reset operator $\mathcal{L}(\rho_{\text{AI}}(t))$ given by

$$H_{\text{cond}} = \frac{1}{\Delta t} \int_t^{t+\Delta t} dt' \langle 0 | H_{\text{I}}(t') | 0 \rangle - \frac{i}{\hbar \Delta t} \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' \langle 0 | H_{\text{I}}(t') H_{\text{I}}(t'') | 0 \rangle \quad (57)$$

and

$$\mathcal{L}(\rho_{\text{AI}}(t)) = \frac{1}{\hbar^2 \Delta t} \int_t^{t+\Delta t} dt' \int_t^{t+\Delta t} dt'' \text{Tr}_{\text{field}} \left(H_{\text{I}}(t') | 0 \rangle \rho_{\text{AI}}(t) \langle 0 | H_{\text{I}}(t'') \right). \quad (58)$$

In the case of an environment that monitors the spontaneous emission of photons, the non-Hermitian Hamiltonian H_{cond} describes the time evolution of the atom under the condition of no photon emission, while the reset operator $\mathcal{L}(\rho_{\text{AI}}(t))$ denotes the un-normalised state of the atom in the case of a photon emission at a time t [4–6].

B. The interaction Hamiltonian $H_{\text{I}}(t)$

Before we can evaluate Eqs. (57) and (58), we need to derive the interaction Hamiltonian $H_{\text{I}}(t)$ in Eq. (50). Suppose $|1\rangle$ denotes the ground state of the atom and $|2\rangle$ is its excited state with energy $\hbar\omega_0$. Then the free Hamiltonian of the atom can be written as

$$H_{\text{atom}} = \hbar\omega_0 |2\rangle\langle 2|. \quad (59)$$

The system Hamiltonian H_{sys} of the electromagnetic field and the semi-transparent mirror can be found in Eq. (45). Moreover, the atom-field interaction Hamiltonian H_{int} equals

$$H_{\text{int}} = e \mathbf{D} \cdot \mathbf{E}_{\text{mirr}}(\mathbf{r}) \quad (60)$$

in the usual dipole approximation. Here e is the charge of a single electron, $\mathbf{E}_{\text{mirr}}(\mathbf{r})$ is the observable of the electric field at the position \mathbf{r} of the atom, while

$$\mathbf{D} = \mathbf{D}_{12} \sigma^- + \mathbf{D}_{12}^* \sigma^+ \quad (61)$$

is the atomic dipole moment. Here \mathbf{D}_{12} is a complex vector and $\sigma^+ = |2\rangle\langle 1|$ and $\sigma^- = |1\rangle\langle 2|$ are the atomic raising and lowering operators respectively. Using these equations, transferring the total Hamiltonian H into the

above specified interaction picture and applying the rotating wave approximation finally yields the interaction Hamiltonian

$$H_{\text{I}}(t) = \frac{ie}{4\pi} \sum_{\lambda=1,2} \int_{\mathbb{R}^3} d^3\mathbf{k} \sqrt{\frac{\hbar\omega}{\pi\epsilon}} e^{-i(\omega-\omega_0)t} \times \left[\left(\frac{1}{\eta_a} a_{\mathbf{k}\lambda} + \frac{t_b}{\eta_b} b_{\mathbf{k}\lambda} \right) e^{i\mathbf{k}\cdot\mathbf{r}} \left(\mathbf{D}_{12}^* \cdot \hat{\mathbf{e}}_{\mathbf{k}\lambda} \right) - \frac{r_a}{\eta_a} a_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot\tilde{\mathbf{r}}} \left(\tilde{\mathbf{D}}_{12}^* \cdot \hat{\mathbf{e}}_{\mathbf{k}\lambda} \right) \right] \sigma^+ + \text{H.c.}, \quad (62)$$

if we place the atom on the right hand side of the mirror, where its x -coordinate is positive. The atomic dipole moment $\tilde{\mathbf{D}}_{12}$ differs from \mathbf{D}_{12} only by the sign of its x -component. From classical electrodynamics we know that $\tilde{\mathbf{D}}_{12}$ is the dipole moment of the mirror image of the atom.

C. Atomic master equations

In the following we use the Hamiltonian $H_{\text{I}}(t)$ in Eq. (62) to obtain the atomic master equations which we introduced in Eq. (56). Using quantum optical standard approximations, ignoring atomic level shifts and proceeding as described in Apps. C and D, we find that

$$\dot{\rho}_{\text{AI}} = \Gamma_{\text{mirr}} \left[\sigma^- \rho_{\text{AI}} \sigma^+ - \frac{1}{2} \sigma^+ \sigma^- \rho_{\text{AI}} - \frac{1}{2} \rho_{\text{AI}} \sigma^+ \sigma^- \right]. \quad (63)$$

In the case of an atom on the right-hand side of the mirror, the spontaneous decay rate Γ_{mirr} equals

$$\Gamma_{\text{mirr}} = \left[\frac{1}{\eta_a^2} (1 + r_a^2) + \frac{t_b^2}{\eta_b^2} - \frac{3r_a}{\eta_a^2} (1 - \mu) \frac{\sin(2k_0x)}{2k_0x} - \frac{3r_a}{\eta_a^2} (1 + \mu) \left(\frac{\cos(2k_0x)}{(2k_0x)^2} - \frac{\sin(2k_0x)}{(2k_0x)^3} \right) \right] \Gamma_{\text{free}}. \quad (64)$$

Here $k_0 = \omega_0/c$ and the constant μ denotes the orientation of the atomic dipole moment,

$$\mu = \|\hat{\mathbf{D}}_{12} \cdot \hat{\mathbf{x}}\|^2 \quad (65)$$

with $\hat{\mathbf{x}}$ and $\hat{\mathbf{D}}_{12}$ being unit vectors in the direction of x and in the direction of the atomic dipole moment. The master equation of an atom in front of a semi-transparent mirror is of exactly the same form as the master equation of an atom in free space, but the presence of a mirror strongly alters the spontaneous decay rate.

For atom-mirror distances x much larger than the wavelength λ_0 of the emitted light, we have $k_0x \gg 1$ and Eq. (64) simplifies to

$$\Gamma_{\text{mirr}} = \left[\frac{1}{\eta_a^2} (1 + r_a^2) + \frac{t_b^2}{\eta_b^2} \right] \Gamma_{\text{free}}. \quad (66)$$

Moreover, proceeding as before but placing the atom on the left-hand side of the mirror, where x is negative,

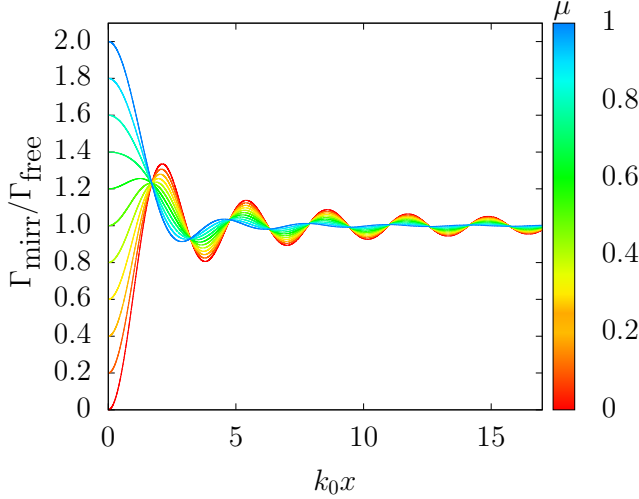


FIG. 5. [Colour online] Dependence of the spontaneous decay rate Γ_{mirr} in Eq. (69) of an atom in front of a perfect mirror on the atom-mirror distance x for different dipole orientations μ . For $\mu = 0$, the atomic dipole moment \mathbf{D}_{12} is parallel to the mirror surface (c.f. Eq. (65)). For $\mu = 1$, the dipole moment \mathbf{D}_{12} points towards the mirror surface.

yields the spontaneous decay rate

$$\Gamma_{\text{mirr}} = \left[\frac{1}{\eta_b^2} (1 + r_b^2) + \frac{t_a^2}{\eta_a^2} \right] \Gamma_{\text{free}} \quad (67)$$

for $k_0x \gg 1$. For simplicity we assume in the following, that the semi-transparent mirror which we consider here is surrounded by free space, i.e. the mirror borders on both sides on a medium with permittivity ε . When this applies, the spontaneous decay rates in Eqs. (66) and (67) should both coincide with the free-space decay rate Γ_{free} in Eq. (1). Taking this into account, we find that the normalisation factors η_a and η_b are such that

$$\eta_a^2 = \frac{(1 + r_a^2)(1 + r_b^2) - (t_a t_b)^2}{1 + r_b^2 - t_b^2}, \quad \eta_b^2 = \frac{(1 + r_a^2)(1 + r_b^2) - (t_a t_b)^2}{1 + r_a^2 - t_a^2}. \quad (68)$$

In free space, we have $r_a, r_b = 0$ and $t_a, t_b = 1$. Taking this into account and using Eqs. (66) and (67), we find that Γ_{mirr} indeed equals Γ_{free} in this case when Eq. (38) applies, as pointed out in Sec. IIID. However, in general, the atomic decay rate Γ_{mirr} depends on the transmission and reflection rates of the mirror and on the atomic parameters x and μ . To gain some intuition for the results of this section, we now have a closer look at some concrete examples. Generalisation of the above results to the situation where the different sides of the mirror couple to media with different permittivities ε is relatively straightforward.

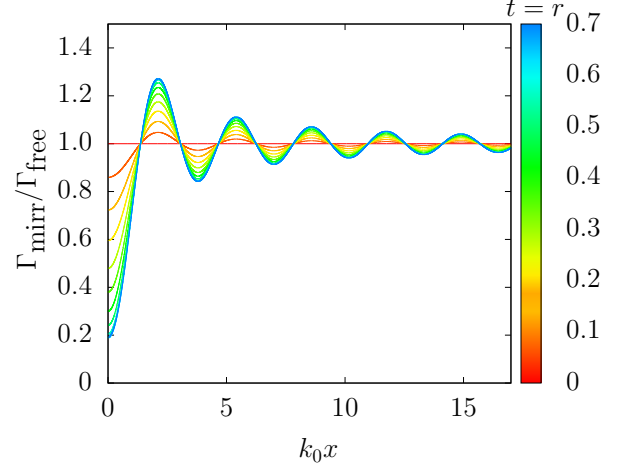


FIG. 6. [Colour online] Dependence of the spontaneous decay rate Γ_{mirr} of an atom in front of a symmetric mirror on the distance x for different values of r (c.f. Eq. (71)). For simplicity we assume here that $r = t$ and $\mu = 0$. The case $r = 0$ corresponds to a completely absorbing surface, while $r = 1/\sqrt{2}$ corresponds to a beamsplitter.

1. Spontaneous emission in front of a perfect mirror

For example, in the case of a perfect two-sided mirror, we have $r_a, r_b = 1$ and $t_a, t_b = 0$. This implies $\eta_a = \eta_b = \sqrt{2}$. Substituting these parameters into Eq. (64), we obtain the spontaneous decay rate

$$\Gamma_{\text{mirr}} = \left[1 - \frac{3}{2} (1 - \mu) \frac{\sin(2k_0x)}{2k_0x} - \frac{3}{2} (1 + \mu) \left(\frac{\cos(2k_0x)}{(2k_0x)^2} - \frac{\sin(2k_0x)}{(2k_0x)^3} \right) \right] \Gamma_{\text{free}}. \quad (69)$$

This decay rate is the result of a dipole-dipole interaction between the atom and its mirror image. However, as pointed out in App. C, the mirror image and the original atom do not have the same dipole moment. Both dipole moments have a different x component. Hence the interaction between both is different from the dipole-dipole interaction between two atoms with exactly the same dipole moment [46] as previously predicted (see e.g. Refs. [17, 19]). Indeed there are several ways of quantising the electromagnetic field near a perfect mirror which are consistent with the boundary condition in Eq. (11), since this condition only concerns the y and the z component of the electric field, but not all of these approaches yield meaningful results.

Fig. 5 shows the x dependence of the spontaneous decay rate Γ_{mirr} of an atom in front of a perfect mirror for different dipole orientations μ . For distances x of the same order of magnitude as the wavelength λ_0 of the emitted light, the last terms in Eq. (69) are no longer negligible. As a result, Γ_{mirr} depends strongly on x and μ . As one would expect, this dependence is most pro-

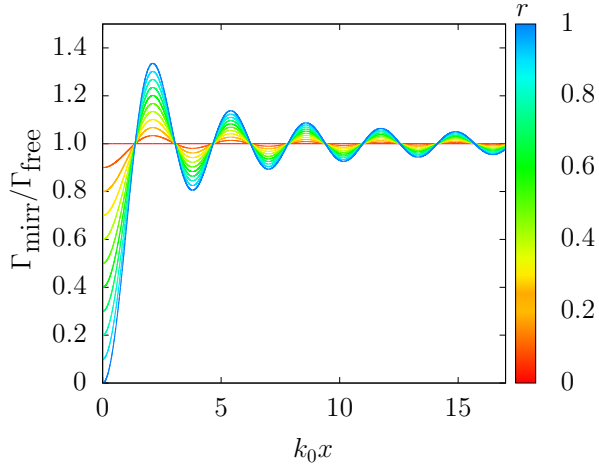


FIG. 7. [Colour online] Dependence of the spontaneous decay rate Γ_{mirr} of an atom in front of a symmetric, non-absorbing semi-transparent mirror on the atom-mirror distance x for different values of r (c.f. Eq. (71)). Again we assume $\mu = 0$, while $t^2 = 1 - r^2$. The case $r = 0$ corresponds again to free space, while $r = 1$ corresponds to a perfect mirror.

nounced and most long-range when $\mu = 0$, i.e. in the case of an atomic dipole moment that is parallel to the mirror surface. In contrast to this, the decay rate Γ_{mirr} approaches Γ_{free} much more quickly when $\mu = 1$. In both cases, we have $\Gamma_{\text{mirr}} = 0$ for $x = 0$, since the electric field amplitude vanishes on the surface of a perfectly conducting mirror (c.f. Eq. (11)).

2. Spontaneous emission in front of a symmetric mirror

In the case of a symmetric mirror with the same transmission and reflection rates on both sides of the mirror, i.e. when

$$t_a, t_b = t \quad \text{and} \quad r_a, r_b = r, \quad (70)$$

the normalisation factors η_a and η_b in Eq. (68) become the same. In this case we have $\eta_a^2 = \eta_b^2 = 1 + 2r^2$. Hence, using Eq. (64), one can show that the spontaneous decay rate Γ_{mirr} of an atom in front of a symmetric mirror equals

$$\begin{aligned} \Gamma_{\text{mirr}} = & \left[1 - 3r(1 - \mu) \frac{1 + r^2 - t^2}{(1 + r^2)^2 - t^4} \cdot \frac{\sin(2k_0x)}{2k_0x} \right. \\ & - 3r(1 + \mu) \frac{1 + r^2 - t^2}{(1 + r^2)^2 - t^4} \\ & \left. \times \left(\frac{\cos(2k_0x)}{(2k_0x)^2} - \frac{\sin(2k_0x)}{(2k_0x)^3} \right) \right] \Gamma_{\text{free}}. \quad (71) \end{aligned}$$

Again, Γ_{mirr} depends strongly on μ and r for relatively short atom-mirror distances x , but tends to the free-space decay rate Γ_{free} , when $k_0x \gg 1$. This is illustrated in Figs. 6 and 7 which show the spontaneous decay rate Γ_{mirr} as a function of x for different values of r and t , while $\mu = 0$.

3. Spontaneous emission in front of a highly-absorbing surface

Finally, we have a closer look at a mirror that absorbs all incoming light. This case corresponds to $r_a = 0$ which yields (c.f. Eq. (64))

$$\Gamma_{\text{mirr}} = \Gamma_{\text{free}}, \quad (72)$$

independent of the atom-mirror distance x and the orientation of the atomic dipole moment μ . As one would expect, an atom near an absorbing medium does not see the mirror and decays exactly as it would in free space. This is illustrated in Figs. 6 and 7 which both show a flat line for $r_a = 0$.

V. CONCLUSIONS

The main result of this paper is the quantisation of the electromagnetic field surrounding a semi-transparent mirror. Using a quantum image detector method, we obtain expressions for the system Hamiltonian H_{sys} and the electric field observable $\mathbf{E}_{\text{mirr}}(\mathbf{r})$ (c.f. Eqs. (45)–(46) with the normalisation constants η_a and η_b given in Eq. (68)). In contrast to H_{sys} , which is independent of the transmission and reflection rates of the mirror, the electric field observable $\mathbf{E}_{\text{mirr}}(\mathbf{r})$ depends strongly on these rates, which we denote t_a, t_b and r_a, r_b . The possible absorption of light in the mirror surface is explicitly taken into account, i.e. the squares of the absorption and transmission coefficients, $r_a^2 + t_a^2$ and $r_b^2 + t_b^2$, do not have to add up to one. However, for simplicity we assume in this paper that the reflection and transmission rates of the mirror do not depend on the frequency and the angle of the incoming light.

Before quantising the electromagnetic field, Sec. II uses classical electrodynamics to discuss the scattering of light on flat surfaces. One way of modelling the scattering process is to assume that incoming wave packets evolve exactly as in free space. i.e. as if the mirror is not there. However, the presence of the mirror changes how and where electric field amplitudes are measured. In general, a detector at a fixed position in front of the mirror observes a superposition of electric field amplitudes. These are associated with incoming, reflected and transmitted waves and need to be normalised accordingly. Adopting this point of view when deriving the observables of the electromagnetic field near a semi-transparent mirror, we find that the system Hamiltonian H_{sys} is the sum of two free-space field Hamiltonians H_{free} and acts on the product of two free-space Hilbert spaces. Moreover, the electric field observable $\mathbf{E}_{\text{mirr}}(\mathbf{r})$ is a superposition of several free-space electric field contributions. To correctly normalise these contributions, we demand that an atom at a relatively large distance away from the mirror has the same spontaneous decay rate as an atom in free space. One can easily check that our approach is consistent with Maxwell's equations.

The main difference between our field quantisation scheme and the field quantisation schemes of other authors is that the energy of the mirror surface, i.e. of the mirror images, is explicitly taken into account. For example, as pointed out in Sec. IIIB, when placing a wave packet in front of a perfect mirror, half of the energy of the system belongs to the original wave packet and the other half belongs to its mirror image and is stored in mirror surface charges. In general, there is a difference between the system Hamiltonian H_{sys} and the Hamiltonian H_{field} of the electromagnetic field surrounding the semi-transparent mirror. Energy can flow from the field onto the mirror surface and back. In the case of absorption, the interaction with the mirror surface reduces the energy of incoming wave packets without destroying their coherence properties and their shape.

Finally, Sec. IV studies the spontaneous emission of an atom in front of a semi-transparent mirror. In good agreement with what one would expect, the presence of a highly absorbing surface does not alter the spontaneous decay rate of an atom. In case of a perfectly-reflecting mirror, our approach reproduces the well-known sub and super-radiance of an atom which has already been verified experimentally [21]. In general, the spontaneous decay rate Γ_{mirr} of an atom in front of a semi-transparent mirror depends in a relatively complex way on system parameters, especially transmission and reflection rates of the mirror surface and the corresponding free space decay rates Γ_{free} in the adjacent media (c.f. Eq. (64) for more details). We expect that the results derived in this paper will have a wide range of applications in quantum technology.

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Appendix A: Calculation of H_{field} in free space

Substituting the field observables $E_{\text{free}}(x)$ and $B_{\text{free}}(x)$ in Eq. (24) into Eq. (21), we find that

$$H_{\text{field}} = -\frac{\hbar}{8\pi} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \sqrt{\omega\omega'} \times \left(e^{ikx} a_k - e^{-ikx} a_k^\dagger \right) \left(e^{ik'x} a_{k'} - e^{-ik'x} a_{k'}^\dagger \right) \times [1 + \text{sign}(kk')] . \quad (\text{A1})$$

To simplify this integral, we first notice that we only obtain a non-zero contribution when k and k' have the

same sign. Moreover, we employ the relation

$$\int_{-\infty}^{\infty} dx e^{\pm ik_0 x} = 2\pi \delta(k_0) , \quad (\text{A2})$$

where k_0 denotes a constant. Taking this into account yields

$$H_{\text{field}} = \int_{-\infty}^{\infty} dk \frac{1}{2} \hbar \omega \left(a_k a_k^\dagger + a_k^\dagger a_k \right) , \quad (\text{A3})$$

which coincides with the harmonic oscillator Hamiltonian H_{free} up to a constant, the so-called zero point energy, as pointed out in Eq. (22).

Appendix B: Calculation of H_{field} for a perfect one-sided mirror

Substituting $E_{\text{mirr}}(x)$ and $B_{\text{mirr}}(x)$ in Eq. (28) into H_{field} in Eq. (29), we now find that

$$H_{\text{field}} = -\frac{\hbar}{8\pi} \int_0^{\infty} dx \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \sqrt{\omega\omega'} \times \left(e^{ikx} \xi_k - e^{-ikx} \xi_k^\dagger \right) \left(e^{ik'x} \xi_{k'} - e^{-ik'x} \xi_{k'}^\dagger \right) \times [1 + \text{sign}(kk')] . \quad (\text{B1})$$

Replacing x by $-x$, k by $-k$ and k' by $-k'$, and taking into account that $\xi_{-k} = -\xi_k$ by definition, one can show that this field Hamiltonian can also be written as

$$H_{\text{field}} = -\frac{\hbar}{8\pi} \int_{-\infty}^0 dx \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \sqrt{\omega\omega'} \times \left(e^{ikx} \xi_k - e^{-ikx} \xi_k^\dagger \right) \left(e^{ik'x} \xi_{k'} - e^{-ik'x} \xi_{k'}^\dagger \right) \times [1 + \text{sign}(kk')] . \quad (\text{B2})$$

Hence we find that

$$H_{\text{field}} = -\frac{\hbar}{16\pi} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \sqrt{\omega\omega'} \times \left(e^{ikx} \xi_k - e^{-ikx} \xi_k^\dagger \right) \left(e^{ik'x} \xi_{k'} - e^{-ik'x} \xi_{k'}^\dagger \right) \times [1 + \text{sign}(kk')] . \quad (\text{B3})$$

Employing again the relation in Eq. (A2) now yields

$$H_{\text{field}} = \int_{-\infty}^{\infty} dk \frac{1}{4} \hbar \omega \left(\xi_k \xi_k^\dagger + \xi_k^\dagger \xi_k \right) \quad (\text{B4})$$

in analogy to Eq. (A3), which coincides with the harmonic oscillator Hamiltonian in Eq. (30) up to a constant.

Appendix C: Calculation of H_{cond} for a semi-transparent mirror

Without restrictions, we consider in the following a coordinate system in which the atomic dipole moment \mathbf{D}_{12}

can be written as

$$\mathbf{D}_{12} = \|\mathbf{D}_{12}\| \begin{pmatrix} d_1 \\ 0 \\ d_3 \end{pmatrix} \quad (\text{C1})$$

with $|d_1|^2 + |d_3|^2 = 1$. In this coordinate system, the dipole moment $\tilde{\mathbf{D}}_{12}$ of the mirror image of the atom equals

$$\tilde{\mathbf{D}}_{12} = \|\mathbf{D}_{12}\| \begin{pmatrix} -d_1 \\ 0 \\ d_3 \end{pmatrix}. \quad (\text{C2})$$

Using this notation and Eq. (62), the conditional Hamiltonian H_{cond} in Eq. (57) becomes

$$\begin{aligned} H_{\text{cond}} = & - \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' \sum_{\lambda=1,2} \int_{\mathbb{R}^3} d^3\mathbf{k} \frac{ie^2\omega}{16\pi^3\epsilon \Delta t} \\ & \times \left[\frac{1}{\eta_a^2} \left\| \left(\mathbf{D}_{12}^* e^{i\mathbf{k}\cdot\mathbf{r}} - r_a \tilde{\mathbf{D}}_{12}^* e^{i\mathbf{k}\cdot\tilde{\mathbf{r}}} \right) \cdot \hat{\mathbf{e}}_{\mathbf{k}\lambda} \right\|^2 \right. \\ & \left. + \frac{t_b^2}{\eta_b^2} \left\| \mathbf{D}_{12}^* \cdot \hat{\mathbf{e}}_{\mathbf{k}\lambda} \right\|^2 \right] e^{-i(\omega-\omega_0)(t'-t'')} \sigma^+ \sigma^-. \quad (\text{C3}) \end{aligned}$$

To simplify this equation, we notice that the polarisation vectors $\hat{\mathbf{e}}_{\mathbf{k}\lambda}$ with $\lambda = 1, 2$ and the unit vector $\hat{\mathbf{k}} = \mathbf{k}/\|\mathbf{k}\|$ form a complete set of basis states in \mathbb{R}^3 which implies that

$$\sum_{\lambda=1,2} \|\mathbf{v} \cdot \hat{\mathbf{e}}_{\mathbf{k}\lambda}\|^2 = \|\mathbf{v}\|^2 - \|\mathbf{v} \cdot \hat{\mathbf{k}}\|^2 \quad (\text{C4})$$

for any vector \mathbf{v} . Moreover, to perform the integration in \mathbf{k} -space, we introduce the polar coordinates $(\omega, \varphi, \vartheta)$ such that

$$\mathbf{k} = \frac{\omega}{c} \begin{pmatrix} \cos(\vartheta) \\ \cos(\varphi) \sin(\vartheta) \\ \sin(\varphi) \sin(\vartheta) \end{pmatrix} \quad (\text{C5})$$

and

$$\int_{\mathbb{R}^3} d^3\mathbf{k} = \int_0^\infty d\omega \int_0^\pi d\vartheta \int_0^{2\pi} d\varphi \frac{\omega^2}{c^3} \sin(\vartheta). \quad (\text{C6})$$

Combining the above equations, performing the φ integration and introducing two new variables, $s = \cos(\vartheta)$ and $\xi = t' - t''$, yields

$$\begin{aligned} H_{\text{cond}} = & - \int_t^{t+\Delta t} dt' \int_0^{t'-t} d\xi \int_0^\infty d\omega \int_{-1}^1 ds \frac{ie^2\omega^3 \|\mathbf{D}_{12}\|^2}{8\pi^2\epsilon c^3 \Delta t} \\ & \times \left[\frac{1}{\eta_a^2} \left(|d_1|^2 (1 + r_a^2 + 2r_a \cos(2\omega xs/c)) (1 - s^2) \right. \right. \\ & \left. \left. + \frac{1}{2} |d_3|^2 (1 + r_a^2 - 2r_a \cos(2\omega xs/c)) (1 + s^2) \right) \right. \\ & \left. + \frac{t_b^2}{\eta_b^2} \left(|d_1|^2 (1 - s^2) + \frac{1}{2} |d_3|^2 (1 + s^2) \right) \right] \\ & \times e^{-i(\omega-\omega_0)\xi} \sigma^+ \sigma^-. \quad (\text{C7}) \end{aligned}$$

To perform the integration over ξ , we extend the ξ -integral from $t' - t$ to infinity. This is well justified when $t' - t \sim \Delta t \gg 1/\omega_0$, as is, in general the case. Next we take into account that

$$\int_0^\infty d\xi e^{-i(\omega-\omega_0)\xi} = \pi \delta(\omega - \omega_0) + \frac{i}{\omega - \omega_0}. \quad (\text{C8})$$

Calculating the remaining integrals and ignoring an atomic level shift, which can be absorbed into the free Hamiltonian, finally yields the conditional Hamiltonian H_{cond}

$$H_{\text{cond}} = -\frac{i}{2} \hbar \Gamma_{\text{mirr}} \sigma^+ \sigma^- \quad (\text{C9})$$

with the spontaneous decay rate Γ_{mirr} given in Eq. (64).

Appendix D: Calculation of $\mathcal{L}(\rho_{\text{I}}(t))$ for a semi-transparent mirror

Substituting Eq. (62) into Eq. (58), one can moreover show that $\mathcal{L}(\rho_{\text{AI}}(t))$ equals

$$\begin{aligned} \mathcal{L}(\rho_{\text{AI}}(t)) = & \frac{1}{\Delta t} \int_t^{t+\Delta t} dt' \int_t^{t+\Delta t} dt'' \int_{\mathbb{R}^3} d^3\mathbf{k} \sum_{\lambda=1,2} |g_{\mathbf{k}\lambda}|^2 \\ & \times \left[\frac{1 + r_a^2}{\eta_a^2} - \frac{2r_a}{\eta_a^2} \cos(\mathbf{k} \cdot (\mathbf{r} - \tilde{\mathbf{r}})) + \frac{t_b^2}{\eta_b^2} \right] \\ & \times e^{i(\omega-\omega_0)(t'-t'')} \sigma^- \rho_{\text{AI}}(t) \sigma^+. \quad (\text{D1}) \end{aligned}$$

In the following, we simplify this expression using the same approximations as in App. C. Substituting Eqs. (C4)–(C6) into this equation and taking into account that

$$\int_{t'-(t+\Delta t)}^{t'-t} d\xi e^{i(\omega-\omega_0)\xi} = 2\pi \delta(\omega - \omega_0) \quad (\text{D2})$$

to a very good approximation, finally yields

$$\mathcal{L}(\rho_{\text{AI}}(t)) = \Gamma_{\text{mirr}} \sigma^- \rho_{\text{AI}}(t) \sigma^+ \quad (\text{D3})$$

with Γ_{mirr} as in Eq. (64).

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- [1] M. Planck, *On the law of the energy distribution in the normal spectrum*, Ann. Phys. **4**, 553 (1901).
- [2] G. S. Agarwal, *Quantum statistical theories of spontaneous emission and their relation to other approaches*, Springer Tracts in Modern Physics; Quantum Optics, Vol. **70** (Springer Verlag Berlin, 1974).
- [3] A. Stokes, A. Kurcz, T. P. Spiller and A. Beige, *Extending the validity range of quantum optical master equations*, Phys. Rev. A **85**, 053805 (2012).
- [4] G. C. Hegerfeldt, *How to reset an atom after a photon detection: Applications to photon-counting processes*, Phys. Rev. A **47**, 449 (1993).
- [5] J. Dalibard, Y. Castin, and K. Mølmer, *Wave-function approach to dissipative processes in quantum optics*, Phys. Rev. Lett. **68**, 580 (1992).
- [6] H. Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics, Volume **18** (Springer-Verlag Berlin, 1993).
- [7] H. Morawitz, *Self-coupling of a two-level system by a mirror*, Phys. Rev. **187**, 1792 (1969).
- [8] K. H. Drexhage, *Influence of a dielectric interface on fluorescence decay time*, J. Lum. **1**, 693-701 (1970).
- [9] P. Stehle, *Atomic radiation in a cavity*, Phys. Rev. A **2**, (1970).
- [10] P. W. Milonni and P. L. Knight, *Spontaneous emission between mirrors*, Opt. Comm. **9**, (1973).
- [11] D. Kleppner, *Inhibited spontaneous emission*, Phys. Rev. Lett. **47**, (1981).
- [12] J. M. Wylie and J. E. Sipe, *Quantum electrodynamics near an interface*, Phys. Rev. A **30**, 1185 (1984).
- [13] H. F. Arnoldus and T. F. George, *Spontaneous decay and atomic fluorescence near a metal surface or an absorbing dielectric*, Phys. Rev. A **37** (1988).
- [14] D. Meschede, W. Jhe and E. A. Hinds, *Radiative properties of atoms near a conducting plane: An old problem in a new light*, Phys. Rev. A **41**, 1587 (1989).
- [15] K. E. Drabe, G. Cnossen and D. A. Wiersma, *Localization of spontaneous emission near a mirror*, Opt. Comm. **73**, (1989).
- [16] H. F. Arnoldus, H. F. George and C. I. Um, *Spontaneous decay near a metal surface*, J. Kor. Phys. Soc. **26** (1993).
- [17] R. Matloob, *Radiative properties of an atom in the vicinity of a mirror*, Phys. Rev. A **62**, 022113 (2000).
- [18] U. Dörner and P. Zoller, *Laser driven atoms in half-cavities*, Phys. Rev. A **62**, 022113, (2000).
- [19] A. Beige, J. K. Pachos and H. Walther, *Spontaneous emission of an atom near a mirror*, Phys. Rev. A **66**, 063801 (2002).
- [20] D. Scerri, T. S. Santana, B. D. Gerardot and E. M. Gauger, *Method of images applied to driven solid-state emitters*, arXiv:1701.04432v1 (2017).
- [21] J. Eschner, C. Raab, F. Schmidt-Kaler and R. Blatt, *Light interference from single atoms and their mirror images*, Nature **413**, 495 (2001).
- [22] M. Babiker and G. Barton, *Quantum frequency shifts near a plasma surface*, J. Phys. A **9**, 129 (1976).
- [23] L. Knöll, W. Vogel and D.-G. Welsch, *Action of passive, lossless optical systems in quantum optics*, Phys. Rev. A **36**, 3803, (1988).
- [24] R. J. Glauber and M. Lewenstein, *Quantum optics of dielectric media*, Phys. Rev. A **43**, 467 (1991).
- [25] B. Huttner and S. M. Barnett, *Quantization of the electromagnetic field in dielectrics*, Phys. Rev. A **46**, 4306 (1992).
- [26] S. T. Wu and C. Eberlein, *Quantum electrodynamics of an atom in front of a non-dispersive dielectric half-space*, Proc. Roy. Soc. **455**, 2487 (1999).
- [27] S. M. Dutra and G. Nienhuis, *Quantized mode of a leaky cavity*, Phys. Rev. A **62**, 063805 (2000).
- [28] L. G. Sutorp and M. Wubs, *Field quantization in inhomogeneous absorptive dielectrics*, Phys. Rev. A **70**, 013816 (2004).
- [29] T. G. Philbin, *Canonical quantization of macroscopic electromagnetism*, New J. Phys. **12**, 123008 (2010).
- [30] T. M. Barlow, R. Bennett and A. Beige, *A master equation for a two-sided optical cavity*, J. Mod. Opt. **62**, S11 (2015).
- [31] M. Liscidini, L. G. Helt, and J. E. Sipe, *Asymptotic fields for a Hamiltonian treatment of nonlinear electromagnetic phenomena*, Phys. Rev. A **85**, 013833 (2012).
- [32] B. J. Dalton, S. M. Barnett and P. L. Knight, *Quasi mode theory of macroscopic canonical quantization in quantum optics and cavity quantum electrodynamics*, J. Mod. Opt. **46**, 1315 (1999).
- [33] M. J. Collett and C. W. Gardiner, *Squeezing of intracavity and travelling-wave light fields produced in parametric amplification*, Phys. Rev. A **30**, 1386 (1984).
- [34] C. W. Gardiner and M. J. Collett, *Input and output in damped quantum systems: Quantum stochastic differential equations and the master equation*, Phys. Rev. A **31**, 3761 (1985).
- [35] S. Fan, S. E. Kocabaş and J. T. Shen, *Input-output formalism for few-photon transport in one-dimensional nanophotonic waveguides coupled to a qubit*, Phys. Rev. A **82**, 063821 (2010).
- [36] Y. L. Lim and A. Beige, *Postselected multiphoton entanglement through Bell-multiport beam splitters*, Phys. Rev. A **71**, 062311 (2005).
- [37] S. Scheel and S. Y. Buhmann, *Macroscopic quantum electrodynamics - concepts and applications*, Acta Phys. Slov. **58**, 675 (2008).
- [38] M. Khanbekyan, L. Knöll, D.-G. Welsch, A. A. Semenov, and W. Vogel, *QED of lossy cavities: Operator and quantum-state input-output relations*, Phys. Rev. A **72**, 053813 (2005).
- [39] R. Bennett, T. M. Barlow and A. Beige, *A physically motivated quantization of the electromagnetic field*, Eur. J. Phys. **37**, 14001 (2016).
- [40] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, 1998).
- [41] L. A. Clark, A. Stokes and A. Beige, *Quantum-enhanced metrology with the single-mode coherent states of an optical cavity inside a quantum feedback loop*, Phys. Rev. A **94**, 023840 (2016).
- [42] E. S. Kyoseva, A. Beige, and L. C. Kwek, *Coherent cavity networks with complete connectivity*, New J. Phys. **14**, 023023 (2012).
- [43] S. Haroche, *Nobel Lecture: Controlling photons in a box and exploring the quantum to classical boundary*, Rev. Mod. Phys. **85**, 1083 (2013).
- [44] C. Schön and A. Beige, *Analysis of a two-atom double-slit experiment based on environment-induced measurements*,

Phys. Rev. A **64**, 023806 (2001).

- [45] W. H. Zurek, *Decoherence, einselection, and the quantum origins of the classical*, Rev. Mod. Phys. **75**, 715 (2003).
- [46] A. Beige and G. C. Hegerfeldt, *Transition from anti-bunching to bunching for two dipole-interacting atoms*,

Phys. Rev. A **58**, 4133 (1998).